

**HW Four, MTH 418, Spring 2016**

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**QUESTION 1.** (i) Let  $n \geq 4$  Find  $\kappa(C_n)$  and  $\kappa'(C_n)$ .(ii) Let  $H$  be a 2-regular connected graph with  $n$  vertices where  $n \geq 3$ . In at most 6 lines, convince me that  $H = C_n$ .(iii) Let  $H$  be a 4-regular connected graph and assume that  $\kappa(H) = 4$ . In one line, find  $\kappa'(H)$ ? and verify your answer.(iv) Let  $H = Q_k$  where  $k \geq 3$ . What is  $\kappa(H)$  and  $\kappa'(H)$ ? Convince me that your claim is correct, in few lines, by showing me how you choose your cut-edge set and cut-vertex set for the graphs  $K_3, K_4$ (v) Let  $H$  be a connected graph such that  $\kappa(H) = 21$ . Convince me that  $girth(H) < \infty$ .(vi) Give me an example of an Eulerian graph such that  $\kappa'(H) = 6$ .(vii) Give me an example of an Eulerian graph that is not an Eulerian circuit but  $\kappa(H) = 3$ .(viii) Let  $H$  and  $D$  be Hamiltonian graphs such that each is of order 3. Convince me that  $H \times D$  is Hamiltonian by constructing a Hamiltonian cycle of  $H \times D$ . Is there anything special about 3?(ix) Let  $H$  be an 8-regular connected graph of order  $n$  where  $n$  is odd. Find  $\chi'(H)$ . Convince me that your claim is correct.

(x) Give me an example of complete bipartite graph that has a maximum matching set but it has no perfect matching set.

(xi) Give me an example of complete bipartite graph that has perfect matching set.

(xii) Construct a perfect matching set for  $Q_3$ .**Due date: Sunday April 24, 2016 Faculty information**